

May 2022 – Higher Tier Solutions

1st

$$M = 2(10) - 3(-4)^2 = 20 - 3(16) = 20 - 48 = -28$$

2nd

$$p = \frac{3r^2 - n}{5}$$

$$5p = 3r^2 - n$$

$$5p + n = 3r^2$$

$$\frac{5p + n}{3} = r^2$$

$$r = \pm \sqrt{\frac{5p + n}{3}}$$

3rd

$$\frac{1}{6x^2 + 5x - 4} \div \frac{1}{9x^2 - 16}$$

$$\frac{1}{(3x + 4)(2x - 1)} \div \frac{1}{(3x + 4)(3x - 4)}$$

$$\frac{1}{(3x + 4)(2x - 1)} \times \frac{(3x + 4)(3x - 4)}{1} = \frac{3x - 4}{2x - 1} \quad a = 3, \quad b = -4, \quad c = 2, \quad d = -1$$

4th

$$32^{1-x} = 8^{3x+5}$$

$$(2^5)^{1-x} = (2^3)^{3x+5}$$

$$2^{5-5x} = 2^{9x+15}$$

$$5 - 5x = 9x + 15$$

$$5 = 14x + 15$$

$$-10 = 14x$$

$$-\frac{10}{14} = x \left(-\frac{5}{7}\right)$$

5th

$$r = \frac{3m - 5}{7 - 2m}$$

$$r(7 - 2m) = 3m - 5$$

$$7r - 2mr = 3m - 5$$

$$7r + 5 = 3m + 2mr$$

$$7r + 5 = m(3 + 2r)$$

$$m = \frac{7r + 5}{3 + 2r}$$

6th

$$32.715 \leq c < 32.725$$

$$3205 \leq d < 3215$$

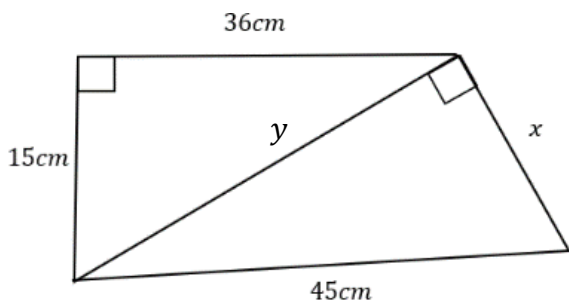
$$\frac{32.715}{\sqrt{3215}} < b < \frac{32.725}{\sqrt{3205}}$$

$$0.576974 < b < 0.5780503$$

$$b = 0.58 \text{ (2 s.f.)}$$

After this the two values differ

7th / 8th



$$15^2 + 36^2 = y^2$$

$$225 + 1296 = y^2$$

$$1521 = y^2$$

$$y = 39\text{cm}$$

$$x^2 + 39^2 = 45^2$$

$$x^2 = 504$$

$$x = 6\sqrt{14}\text{cm or } 22.45\text{cm}$$

9th

(a) 65 : 180

$$1 : \frac{36}{13} \quad (1 : 2.769)$$

(b) Length = $90 \times \frac{36}{13} = \frac{3240}{13}\text{cm or } 249.23\text{cm}$

10th

$$4 \times 10 \times 10 \times 2 = 800$$

11th

$$\begin{pmatrix} 3a + 5 \\ 5b \end{pmatrix} + \begin{pmatrix} 2b - 1 \\ 5 - 4a \end{pmatrix} = \begin{pmatrix} 9 \\ 52 \end{pmatrix}$$

$$3a + 2b = 5 \quad (1)$$

$$-4a + 5b = 47 \quad (2)$$

$$(1) \times 4$$

$$(2) \times -3$$

$$12a + 8b = 20 \quad (3)$$

$$12a - 15b = -141 \quad (4)$$

$$(3) - (4)$$

$$23b = 161 \quad b = 7$$

Sub $b = 7$ into (1)

$$3a + 14 = 5$$

$$3a = -9$$

$$a = -3$$

12th

$$225792 \div 1.12^2 = 180,000$$

13th

$$a) P(R) = \frac{n}{n+9}$$

$$P(R \cap R) = \frac{n}{n+9} \times \frac{(n-1)}{n+8} = \frac{11}{38}$$

$$\frac{n^2 - n}{n^2 + 17n + 72} = \frac{11}{38}$$

$$38n^2 - 38n = 11n^2 + 187n + 792$$

$$27n^2 - 225n - 792 = 0 \quad (\div 9)$$

$$3n^2 - 25n - 88 = 0$$

$$b) 3n^2 - 25n - 88 = 0$$

$$(3n + 8)(n - 11) = 0$$

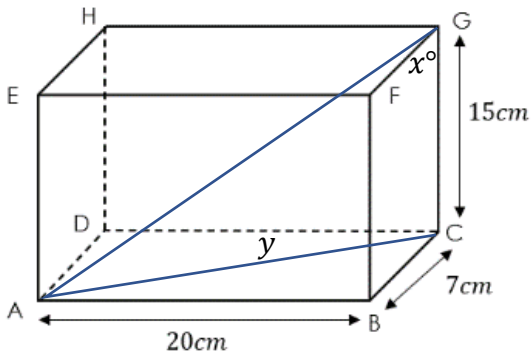
$$n = -\frac{8}{3} \quad n = 11$$

n has to be + & whole so $n = 11$

11 red 9 white

20 in total.

14th / 15th



$$20^2 + 7^2 = y^2$$
$$y = \sqrt{449}$$
$$\tan x = \frac{\sqrt{449}}{15}$$
$$x = 54.7^\circ$$

16th

$$m_{L_1} = \frac{20 - 4}{6 - -14} = \frac{16}{20} = \frac{4}{5}$$

$$m_{L_2} = -\frac{5}{4}$$

Midpoint of AB = (-4, 12)

$$y = -\frac{5}{4}x + c$$

$$12 = -\left(\frac{5}{4}\right)(-4) + c$$

$$12 = 5 + c \quad c = 7$$

$$y = -\frac{5}{4}x + 7$$

17th

(a)

$$5(3x + 1) + 2(5 - 2x) = 15x + 5 + 10 - 4x = 11x + 15$$

(b)

$$(x + 9)(2x - 3) = 2x^2 + 12x - 27$$

18th

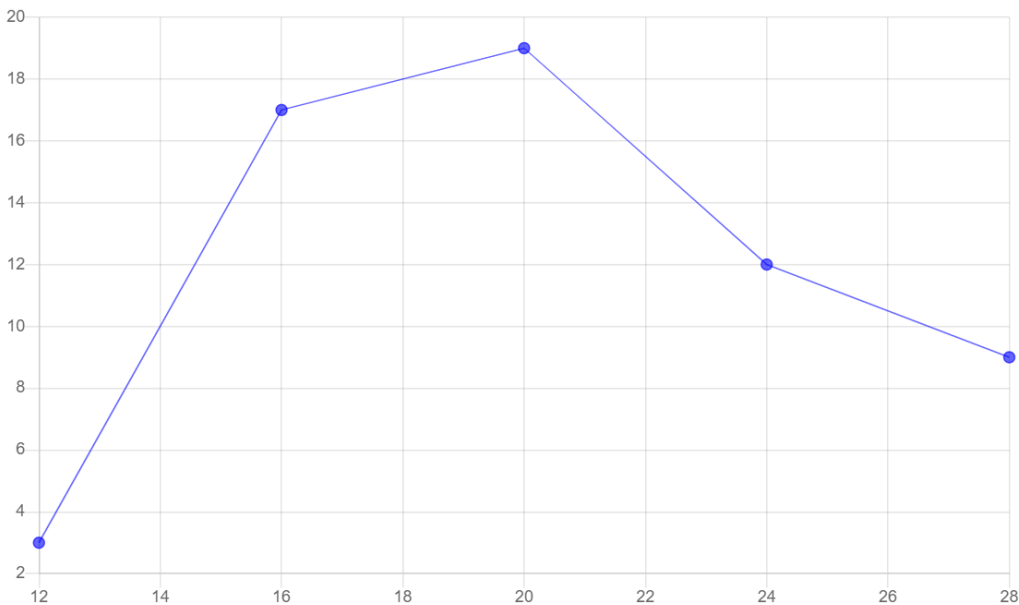
$$12000 \times M^3 = 9345$$

$$M^3 = 0.77875$$

$$M = 0.92$$

$$X = 8\%$$

19th



20th

$$\begin{aligned}
 6 \text{ painters} &= 4 \text{ days} &= 12 \text{ rooms} & \text{(inverse proportion)} \\
 3 \text{ painters} &= 8 \text{ days} &= 12 \text{ rooms} & \text{(direct proportion)} \\
 3 \text{ painters} &= 10 \text{ days} &= 15 \text{ rooms} &
 \end{aligned}$$

21st / 22nd

$$(a) x^3 - 3x - 42 = 0$$

$$f(x) = x^3 - 3x - 42$$

$$f(3) = 27 - 9 - 42 = -24$$

$$f(4) = 64 - 12 - 42 = 10$$

Change of sign so solution between 3 and 4

(b)

$$x^3 - 3x = 42$$

$$x(x^2) = 42 + 3x$$

$$x^2 = \frac{42 + 3x}{x}$$

$$x = \sqrt{\frac{42 + 3x}{x}}$$

(c)

$$x_0 = 3.5$$

$$x_1 = \sqrt{15}$$

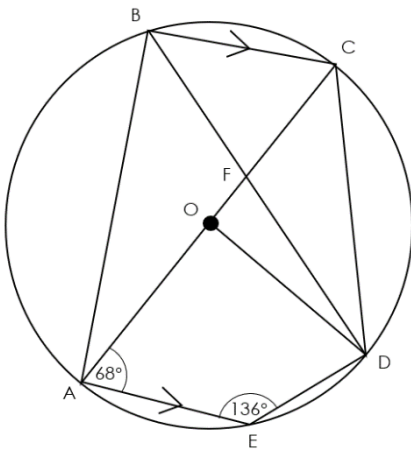
$$x_2 = 3.7208$$

$$x_3 = 3.78$$

23rd

$$P(R R' R') = \frac{7}{16} \times \frac{9}{15} \times \frac{8}{14} = \frac{504}{3360}$$

$$P(\text{one red}) = 3 \times \frac{504}{3360} = \frac{1512}{3360}$$

24th

$ABD = 44$ (opposite angles in cyclic quadrilateral add to 180)

$ABC = 90$ (angle in a semi-circle is a right angle)

$CBF = 90 - 44 = 46$

$BCF = 68$ (alternate angles are equal)

$CFB = 180 - 68 - 46 = 66$ (angles in a triangle add to 180)

25th

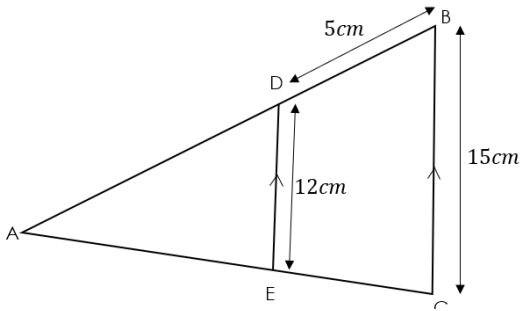
$$\text{Swimming} = \frac{2}{1.4} = \frac{10}{7} \text{ hr}$$

$$\text{Running} = \frac{5}{6} \text{ hr}$$

$$\text{Cycling} = \frac{20}{15} = \frac{4}{3} \text{ hr}$$

$$\text{Total Time} = \frac{10}{7} + \frac{5}{6} + \frac{4}{3} = \frac{151}{42} \text{ hr} = 3\text{h}36\text{m}$$

26th



$$\frac{x + 5}{x} = \frac{5}{4}$$

(a)

Both share angle BAC

$\angle ADE = \angle ABC$ (corresponding angles)

$\angle AED = \angle ACB$ (corresponding angles)

All three angles are same in both triangles therefore they are similar.

(b) Scale Factor = $\frac{15}{12} = \frac{5}{4}$

Let $AD = x$ $AB = x + 5$

$4x + 20 = 5x$ $x = 20$ $AB = 25\text{cm}$

27th

Ann = x

Ben = $3x$

Chris = $3x - 7$

Denise = $x + 13$

$3x - 7 = x + 13$

$2x - 7 = 13$

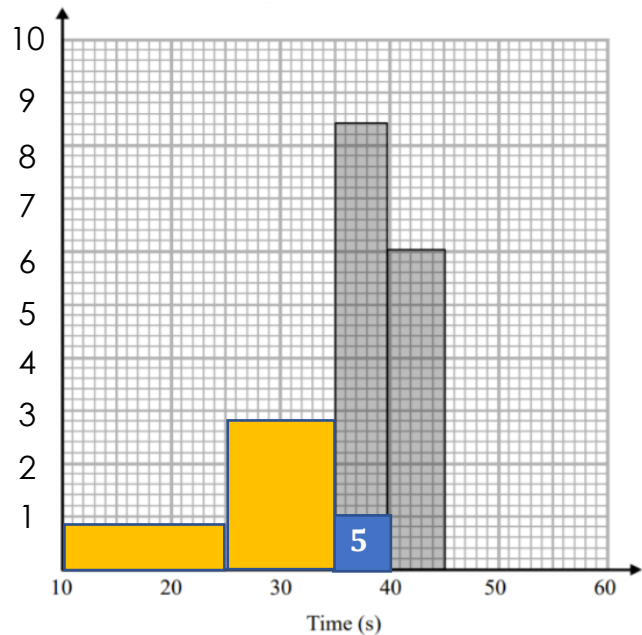
$2x = 20$

$x = 10$

Ann = 10, Ben = 30, Chris and Denise = 23 Total = 86

28th / 29th

Time (s)	Frequency
$10 < t \leq 25$	12
$25 < t \leq 35$	28
$35 < t \leq 40$	42
$40 < t \leq 45$	30
$45 < t \leq 60$	9



Median = 61st time

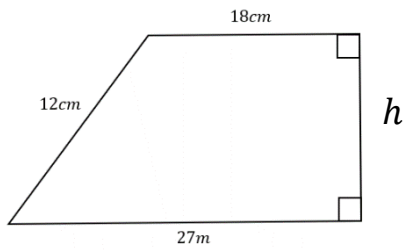
In $35 < t \leq 40$ group

21 into this group – halfway so median = 37.5 seconds

30th

$$y = 2x + 3 \quad \text{--- (1)}$$
$$x^2 + (2x + 3)^2 = 18$$
$$x^2 + 4x^2 + 12x + 9 - 18 = 0$$
$$5x^2 + 12x - 9 = 0$$
$$(5x - 3)(x + 3) = 0$$
$$\left(x = \frac{3}{5} \right) \left(x = -3 \right)$$
$$\left(y = \frac{21}{5} \right) \left(y = -3 \right)$$

31st



$$h^2 + 9^2 = 12^2$$

$$h = 144 - 81 = 63$$

$$h = \sqrt{63}$$

$$\text{Area} = \frac{1}{2}(18 + 27)(\sqrt{63}) = 178.6 \text{ cm}^2$$