**GCE A level Mathematics (9MA0) – Paper 2**

**Pure Mathematics**

**Summer 2018**

**Summer 2018 student-friendly mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide to good practice, indicating where marks are given for correct answers. As such, it doesn’t show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme.**

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| --- |
| **Guidance on the use of codes within this document** |
| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.  A1 – accuracy mark. This mark is generally given for a correct answer following correct working.  B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.  Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

**Question 1 (Total 6 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | g(5) =  = 7.5 | M1 | This mark is given for a method to find the value of g(5) |
| gg(5) = g(7.5) =  =  = | A1 | This mark is given for the correct answer only (or an equivalent value) |
| (b) | For *x* = 5, g(*x*) = 7.5; as *x* → ∞, g(*x*) > 2.  Range of g = 2 < *y* ≤ 7.5 | B1 | This mark is given for the correct answer only |
| (c) | *y* =  *yx* – 3*y* = 2*x* + 5  *yx* – 2*x* = 3*y* + 5 | M1 | This mark is given for a method to cross-multiply and collect terms in *x* or *y* |
| *x*(*y* – 2) = 3*y* + 5  *x* = | M1 | This mark is given for a fully correct method to find the inverse of g(*x*) |
| Thus g–1(*x*) = , 2 < *x* ≤ 7.5 | A1 | This mark is given for a correct answer using correct notation and stating the domain of g–1(*x*) (the same as the range of g(*x*) |

**Question 2 (Total 5 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | =  +  =  +  *=*  *–*  so  =  +  –  =  +  – | M1 | This mark is given for a method to find a vector expression for |
| =  or 6**i** – 7**j** + 10**k** | A1 | This mark is given for the correct answer only |
| (b) | =  –  42 = (*a* – 2)2 + (5 – 3)2 + (–2 – –4)2 | M1 | This mark is given for a method to find the difference between the lengths of  and |
| 16 = (*a* – 2)2 + 4 + 4  8 = (*a* – 2)2 | M1 | This mark is given for a method to find the value of *a* |
| Since *a* < 0, *a* = 2 – √8 | A1 | This mark is given for the correct answer only |

**Question 3 (Total 5 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | For example, √3 and √12 are both irrational, but √3 × √12 = √36 = 6, so the statement is untrue | M1 | This mark is given for two irrational numbers chosen to start the argument |
| A1 | This mark is given for a fully correct statement |
| (b)(i) |  | B1 | This mark is given for a v-shaped graph symmetrical about the *y­-*axis withthe point (0, 3) indicated |
| M1 | This mark is given for the graph of *y*= ⏐*x* + 3⏐ superimposed on the graph of *y*= ⏐*x*⏐+ 3 |
| (b)(ii) | When *x* ≥ 0, both graphs are equal; when *x* < 0, the graph of of *y*= ⏐*x*⏐+ 3 is always above that for *y*= ⏐*x* + 3⏐ | A1 | This mark is given for a fully correct explanation |

**Question 4 (Total 7 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (i) | =  + | M1 | This mark is given for a start to a method to find the sum |
| = (2 × 8 + 15 × 5) + | M1 | This mark is given for a method to find the value of |
| M1 | This mark is given for a method to find the value of |
| = 728 + 131 070 = 131 798 | A1 | This mark is given for a correct proof supported by working |
| (ii) | *u*1 = , *u*2 = , *u*3 = , … | M1 | This mark is given for deducing the repeating terms of the sequence |
| = 50 ×  + 50 × | M1 | This mark is given for a method to deduce how the sum can be found |
| = 50 × ( + ) = | A1 | This mark is given for the correct answer only (or an equivalent fraction) |

**Question 5 (Total 6 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | f′(*x*) = 6*x*2 + 2*x* | B1 | This mark is given for finding the derivative of f(*x*) |
| *xn* + 1 = *xn* –  = *xn* – | M1 | This mark is given for making a substitution into the Newton-Raphson formula |
| =  =  = | A1 | This mark is given for a rearrangement to lead to a correct proof |
| (b) | *x*2 = | M1 | This mark is given for a substitution to find *x*2 |
| *x*2 = , *x*3 = | A1 | This mark is given for the correct answer only |
| (c) | There is a stationary point at *x* = 0 | B1 | This mark is given for a correct statement |

**Question 6 (Total 6 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | f(2) = –24 + 32 – 18 + 10 = 0 | B1 | This mark is given for the correct answer only |
| f(*x*) = (*x* – 2)(–3*x*2 + 2*x* – 5) | M1 | This mark is given for a deduction that (*x* – 2) is a factor |
| A1 | This mark is given for a correct answer only |
| (b) | −3*y*6 + 8*y*4 − 9*y*2 + 10 = 0  so (*y*2 – 2)(–3*y*2 + 2*x* – 5) = 0  *y*2 – 2 has two real solutions, ±√2  For (–3*y*2 + 2*x* – 5),  *b*2 – 4*ac* = 22 – (4 × –3 × –5) = –56 < 0, no real solutions | M1 | This mark is given for showing that either *y*2 – 2 has two real solutions or that –3*y*2 + 2*x* – 5 has no real solutions |
| A1 | This mark is given for a complete proof to show the equation has two real solutions |
| (c) | 3 solutions | B1 | This mark is given for a correct deduction |

**Question 7 (Total 9 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (i) | sec *x* = | B1 | This mark is given for substituting for sec *x* |
| 4 sin *x* cos *x* = 1 ⇒ 2 sin 2*x* = 1  so sin 2*x* = | M1 | This mark is given for a method to use the identity 2 sin *x* cox *x* = sin 2*x* |
| *x* =  arcsin | M1 | This mark is given for a method to find the values of *x* |
| *x* = , | A1 | This mark is given for the (two) correct answers only |
| (ii) | 5 sin *θ* – 5 cos *θ* = 2  *R*(sin *θ* – *α* ) = 2 | M1 | This mark is given for writing the equation in the form *R*(sin *θ* – *α* ) = 2 |
| *R* = √50  tan *α* = 1 ⇒ *α* = 45° | M1 | This mark is given for finding the values of *R* and *α* |
| sin (*θ* – 45°) = | A1 | This mark is given for finding the value of sin (*θ* – 45°) |
| *θ* = arcsin  + 45° | M1 | This mark is given for a method to find the values of *θ* |
| *θ* = 61.4°, 208.6° | A1 | This mark is given for the correct answers only |

**Question 8 (Total 7 marks)**

|  |  |  |  |
| --- | --- | --- | --- |
| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| (a) | *H* = *Ax*(40 – *x*) | M1 | This mark is given for a correct form to model the trajectory of the rugby ball |
| *x* = 20, *H* = 12  12 = *A* × 20 × (40 – 20) = 12  *A* =  = | M1 | This mark is given for a method to find the value of *A* |
| *H* = *x*(40 – *x*) | A1 | This mark is given for a correct quadratic equation |
| (b) | When *H* = 3,  *x*(40 – *x*) = 3  3*x*(40 – *x*) = 300  120*x* – 3*x*2 = 300  *x*2 – 40*x* + 100 = 0 | M1 | This mark is for a method to find a quadratic equation when *H* = 3 |
| *x* =  = 20 ± √300 | M1 | This mark is given for using the quadratic formula to find values for *x* |
| *x* = 20 + √300 = 37.3 m | A1 | This mark is given for choosing the greater distance |
| (c) | For example:  the ground is horizontal  the ball needs to be kicked from the ground  the ball is modelled as a particle  the horizontal bar needs to be modelled as a line  there is no wind or air resistance on the ball  there is no spin on the ball  there are no obstacles in the trajectory (or path) of the ball  the trajectory of the ball is a perfect parabola | B1 | This mark is given for a correct statement |

**Question 9 (Total 5 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
|  |  | B1 | This mark is given for starting with the correct fraction |
| =  = –sin *θ* + cos *θ* | M1 | This mark is given for an attempt to use the compound angle formula for cos (*θ*+ *h*) |
| A1 | This mark is given for correct use of the compound angle formula for cos (*θ* + *h*) |
| As *h* → 0,  –sin *θ* + cos *θ* → ­–sin *θ*  since  = 1 and  = 0 | M1 | This mark is given for a method to apply the given limits to the gradient of the chord |
| Thus = (cos *θ* ) = –sin *θ* | A1 | This mark is given for a complete proof using correct mathematical language |

**Question 10 (Total 8 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| (a) | ∝ ± | M1 | This mark is given for a mathematical description of the model |
| = | M1 | This mark is given for separating the variables of the differential equation formed |
| *r*2 = ± *k t* + *c* | A1 | This mark is given for a correct integration |
| When *t* = 0, *r* = 5 and  *r*2 = , so *c* =  When *t* = 4, *r* = 3  *r*2 = 4*k* +  9 = 4*k* +  *k*  = – | M1 | This mark is given for a method to find values for *k* and *c* |
| *r*2 = –*k* + | A1 | This mark is given for a fully correct equation to describe the model |
| (b) | *r* = 0 ⇒ 0 = –*t* +  250 – 49*t*  = 0  *t* = 5.102… | M1 | This mark is given for a method to find the value of *t* |
| Time = 5 minutes and 6 seconds | A1 | This mark is given for a correct answer only |
| (c) | For example:  The model does not consider how the mint is sucked  The model does not consider whether the mint is bitten  The model is limited for times up to 5 minutes 6 seconds  The model is not valid for times greater than 5 minutes 6 seconds  The mint may not retain the shape of a sphere (or have uniform radius) as it is being sucked  The model indicates that the radius of the mint is negative after it dissolves  The model does not consider the temperature in the mouth  The model does not consider rate of saliva production  The mint could be swallowed before it dissolves in the mouth | B1 | This mark is given for a correct limitation stated |

**Question 11 (Total 7 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| (a) | 1 + 11*x* – 6*x*2 ≡  *A*(1 – 2*x*)(*x* – 3) + *B*(1 – 2*x*) + *C*(*x* – 3) | M1 | This mark is given for using a correct identity to find *B* and *C* |
| 6 = 2*A*,  *A* = 3 | B1 | This mark is given for comparing *x*2 coefficients to find a value for *A* |
| Substituting *x* = 3 gives  1 + (11 × 3) – (6 × 32) = *B* × (1 – 2 × 6)  –20 = –5*B*  Substituting *x* =  gives  1 + (11 × ) – (6 × 2) = *C* × ( – 3)  5 = –2*C* | M1 | This mark is given for a method to find the values of *B* and *C* |
| *B* = 4, *C* = –2 | A1 | This mark is given for the correct answers only |
| (b) | f(*x*) = 3 + 4(*x* – 3)–1 – 2(1 – 2*x*)–1, *x* > 3 | M1 | This mark is given for re-writing f(*x*) |
| f ′(*x*) = –4(*x* – 3)–2 – 4(1 – 2*x*)–2 | A1 | This mark is given for finding a correct expression for f′(*x*) |
| Since (*x* – 3)2 > 0 and (1 – 2*x*)2 > 0 then f ′(*x*) < 0 and so a decreasing function | A1 | This mark is given for a correct explanation |

**Question 12 (Total 9 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | | **Notes** |
| --- | --- | --- | --- | --- |
| (a) | tan *θ* sin 2*θ* = (2 sin *θ* cos *θ* ) | M1 | | This mark is given for re-writing tan *θ* and 2 sin *θ* |
| = sin *θ* (2 sin *θ* ) | M1 | | This mark is given for cancelling cos *θ* from both sides of the equation |
| = 2 sin2 *θ* = 1 – cos 2*θ* | A1 | | This mark is given for a correct proof showing all steps |
| (b) | (sec2 *x* – 5)(1 – cos 2*x*) = 3 tan *x*(1 – cos 2*x*)  ⇒ *x* = 0 | | B1 | This mark is given for deducing that one solution is *x*= 0 |
| 1 + tan2 *x* – 5 = 3 tan *x* | M1 | | This mark is given for using sec2*x*= tan2*x* and cancelling (1 – cos 2*x*) from both sides of the equation |
| tan2 *x* – 3 tan *x* – 4 = 0 | A1 | | This mark is given for rearranging to make a quadratic to be solved |
| (tan *x* – 4)(tan *x* + 1) = 0 | M1 | | This mark is given for factorising the quadratic equation |
| tan *x* = 4, *x* = 1.326 | A1 | | This mark is given for finding one correct value of *x* |
| tan *x* = –1, *x* = | A1 | | This mark is given for finding one other correct value of *x* |

**Question 13 (Total 10 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
|  | = ln *x* + *x* = ln *x* + 1 | M1 | This mark is given for an attempt to differentiate *y* = *x* ln *x* |
| A1 | This mark is given for correctly differentiating *y* = *x* ln *x* |
| At *x* = e, the gradient of *C* is (ln e + 1) = 2  Thus the gradient of *l* at e = ­–  *y* – e = –(*x* – e)  *y* = 0 ⇒ –e = –(*x* – e) | M1 | This mark is given for a method to find the *x*-coordinate of the point where the normal to *C* at *P*(e, e) meets the *x*-axis |
| *l* meets the *x*-axis at *x* = 3e | A1 | This mark is given for finding the point at which *l* meets the *x*-axis |
| 1  e  3e  *R* = + (e × (3e – e)) | M1 | This mark is given for a method to find the area of *R* (split into two areas) |
| =  + e | M1 | This mark is given for a method to integrate *R* |
| = + e | M1 | This mark is given for a method to integrate *R* |
| A1 | This mark is given for a correct integral of *R* |
| = e2 – e2 – 0 +  + e | M1 | This mark is given for a method to evaluate the integral of *R* |
| = e2 + | A1 | This mark is given for a correct answer only with *A* =  and *B* = |

**Question 14 (Total 10 marks)**

| **Part** | **Working or answer an examiner might expect to see** | **Mark** | **Notes** |
| --- | --- | --- | --- |
| (a) | When *t* = 0, *N* =  = 90 | B1 | This mark is given for the correct answer only |
| (b) | = –900(3 + 7e–0.25*t*)–2(7 × –0.25e–0.25*t*) | M1 | This mark is given for an attempt to find an expression for the rate of growth |
| A1 | This mark is given for correctly finding an expression for the rate of growth |
| 3 + 7e–0.25*t* =  =  =  =  = | M1 | This mark is given for an attempt to rearrange terms to show the rate of growth given |
| = | A1 | This mark is given for a complete proof |
| (c) | Value of *T* is greatest when 300 – 2*N* = 0  so when *N* = 150 | B1 | This mark is given for deducing a value for *N* |
| 150 =  ⇒ 3 + 7e–0.25*t* = 6  ⇒ e–0.25*T* = | M1 | This mark is given for a method to find the value of e–0.25*T* |
| *T* = –4 ln | M1 | This mark is given for a method to find an expression for the value of *T* |
| *T* = 3.4 months | A1 | This mark is given for the correct answer only |
| (d) | Rate of growth becomes negative when  is negative, so *P* = 300 | B1 | This mark is given for the correct answer only |