Exercise 2A

In this exercise use either your calculator or the tables on page 172.

1. $Z \sim N(0,1)$. Find the following probabilities.
   (a) $P(Z < 1.23)$  
   (b) $P(Z \leq 2.468)$  
   (c) $P(Z < 0.157)$  
   (d) $P(Z \geq 1.236)$  
   (e) $P(Z > 2.378)$  
   (f) $P(Z \geq 0.588)$  
   (g) $P(Z > -1.83)$  
   (h) $P(Z \geq -2.057)$  
   (i) $P(Z > -0.067)$  
   (j) $P(Z \leq -1.83)$  
   (k) $P(Z < -2.755)$  
   (l) $P(Z \leq -0.206)$  
   (m) $P(Z < 1.645)$  
   (n) $P(Z \geq 1.645)$  
   (o) $P(Z > -1.645)$  
   (p) $P(Z \leq -1.645)$

2. The random variable $Z$ is distributed such that $Z \sim N(0,1)$. Find the probabilities.
   (a) $P(1.15 < Z < 1.35)$  
   (b) $P(1.111 \leq Z \leq 2.222)$  
   (c) $P(0.387 < Z < 2.418)$  
   (d) $P(0 \leq Z < 1.55)$  
   (e) $P(-1.815 < Z < 2.333)$  
   (f) $P(-0.847 < Z \leq 2.034)$  
   (g) $P(-2.505 < Z < 1.089)$  
   (h) $P(-0.55 \leq Z \leq 0)$  
   (i) $P(-2.82 < Z < -1.82)$  
   (j) $P(-1.749 \leq Z \leq -0.999)$  
   (k) $P(-2.568 < Z < -0.123)$  
   (l) $P(-1.96 \leq Z < 1.96)$  
   (m) $P(-2.326 < Z < 2.326)$  
   (n) $P(|Z| \leq 1.3)$  
   (o) $P(|Z| > 2.4)$

3. The random variable $Z \sim N(0,1)$. In each part, find the value of $s$, $t$, $u$ or $v$.
   (a) $P(Z < s) = 0.6700$  
   (b) $P(Z < t) = 0.8780$  
   (c) $P(Z < u) = 0.9842$  
   (d) $P(Z < v) = 0.8455$  
   (e) $P(Z > s) = 0.4052$  
   (f) $P(Z > t) = 0.1194$  
   (g) $P(Z > u) = 0.0071$  
   (h) $P(Z > v) = 0.2241$  
   (i) $P(Z > s) = 0.9977$  
   (j) $P(Z > t) = 0.9747$  
   (k) $P(Z > u) = 0.8496$  
   (l) $P(Z > v) = 0.5$  
   (m) $P(Z < s) = 0.0031$  
   (n) $P(Z < t) = 0.0142$  
   (o) $P(Z < u) = 0.0468$  
   (p) $P(Z < v) = 0.4778$  
   (q) $P(-s < Z < s) = 0.90$  
   (r) $P(-t < Z < t) = 0.80$  
   (s) $P(-u < Z < u) = 0.99$  
   (t) $P(|Z| < v) = 0.50$
Exercise 2B

You are strongly advised to draw rough sketches for these questions.

1. Given that $X \sim N(20, 16)$, find the following probabilities.
   
   - (a) $P(X \leq 26)$
   - (b) $P(X > 30)$
   - (c) $P(X = 17)$
   - (d) $P(X < 13)$

2. Given that $X \sim N(24, 9)$, find the following probabilities.
   
   - (a) $P(X \leq 29)$
   - (b) $P(X > 31)$
   - (c) $P(X = 22)$
   - (d) $P(X < 16)$

3. Given that $X \sim N(50, 16)$, find the following probabilities.
   
   - (a) $P(54 \leq X \leq 58)$
   - (b) $P(40 < X \leq 44)$
   - (c) $P(47 < X < 57)$
   - (d) $P(39 \leq X < 53)$
   - (e) $P(44 \leq X \leq 56)$

4. The random variable $X$ can take negative and positive values. $X$ is distributed normally with mean 3 and variance 4. Find the probability that $X$ has a negative value.

5. The random variable $X$ has a normal distribution. The mean is $\mu$ (where $\mu > 0$) and the variance is $\frac{1}{4} \mu^2$.
   
   - (a) Find $P(X > 1.5\mu)$.
   - (b) Find the probability that $X$ is negative.

6. Given that $X \sim N(44, 25)$, find $s$, $t$, $u$ and $v$ correct to 2 decimal places when
   
   - (a) $P(X \leq s) = 0.9808$
   - (b) $P(X > t) = 0.7704$
   - (c) $P(X \geq u) = 0.0495$
   - (d) $P(X \leq v) = 0.3336$

7. Given that $X \sim N(15, 4)$, find $s$, $t$, $u$, $v$ and $w$ correct to 2 decimal places when
   
   - (a) $P(X \leq s) = 0.9141$
   - (b) $P(X > t) = 0.5746$
   - (c) $P(X > u) = 0.1041$
   - (d) $P(X \leq v) = 0.3924$
   - (e) $P[|X - 15| < w] = 0.9$

8. Given that $X \sim N(35.4, 12.5)$, find the values of $s$, $t$, $u$ and $v$ correct to 1 decimal place when
   
   - (a) $P(X < s) = 0.95$
   - (b) $P(X > t) = 0.9391$
   - (c) $P(X > u) = 0.2924$
   - (d) $P(X < v) = 0.1479$

9. $X$ has a normal distribution with mean 32 and variance $\sigma^2$. Given that the probability that $X$ is less than 33.14 is 0.6406, find $\sigma^2$. Give your answer correct to 2 decimal places.

10. $X$ has a normal distribution, and $P(X > 73.05) = 0.0289$. Given that the variance of the distribution is 18, find the mean.

11. $X$ is distributed normally, $P(X \geq 59.1) = 0.0218$ and $P(X \geq 29.2) = 0.9345$. Find the mean and standard deviation of the distribution, correct to 3 significant figures.

12. $X \sim N(\mu, \sigma^2)$. $P(X \geq 9.81) = 0.1587$ and $P(X \leq 8.82) = 0.0116$. Find $\mu$ and $\sigma$, correct to 3 significant figures.
Exercises 2D

1. State whether the following binomial distributions can or cannot reasonably be approximated by a normal distribution. Write down a brief calculation to justify your conclusion in each case.
   (a) B(50, 0.2)  (b) B(60, 0.1)  (c) B(70, 0.01)  (d) B(30, 0.7)
   (e) B(40, 0.9)

2. A random variable, X, has a binomial distribution with parameters $n = 40$ and $p = 0.3$. Use a suitable approximation, which you should show is valid, to calculate the following probabilities.
   (a) $P(X > 18)$  (b) $P(X < 9)$  (c) $P(X = 15)$  (d) $P(11 < X < 15)$

3. The mass production of a cheap pen results in there being 1 defective pen in 20 on average. Use an approximation, which you should show is valid, to find, in a batch of 300 of these pens, the probability of there being (a) 24 or more defective pens, (b) 10 or fewer defective pens.

4. A fair coin is tossed 18 times.
   (a) Use the binomial distribution to find the probability of obtaining 14 heads.
   (b) Use a normal approximation to find the probability of obtaining 14 heads, and to find the probability of obtaining 14 or more heads. Show that the approximation is valid.

5. In a certain county 12% of people have green eyes. If 50 of these people are inspected, find the probability that
   (a) 12 or more of them have green eyes,
   (b) between 3 and 10 (inclusive) of them have green eyes.
   Show that your approximation is valid.

6. Fred attempts to dial a connection to the internet for his email each day. He is successful on his first attempt 8 times out of 10. Use a normal approximation, showing first that it is valid, to find the probability that Fred is successful on his first attempt at dialling a connection on 36 days or more over a period of 40 days.

7. (a) An unbiased dice is thrown 60 times. Find the probability that a five is obtained on 12 to 18 (inclusive) of these throws.
   (b) In a game two unbiased dice are thrown. A winning score on each throw is a total of 5, 6, 7 or 8. Find the probability of a win on 70 or more throws out of 120 throws.

8. At an election there are two parties, X and Y. On past experience twice as many people voted for party X as for party Y.
   An opinion poll researcher samples 90 voters. Find the probability that 70 or more say they will vote for party X at the next election.
   If 2000 researchers each questioned 90 voters, how many of these researchers would be expected to record ‘70 or more for party X’ results?

9. A manufacturer states that ‘3 out of 4 people prefer my product (Acme) to a competitor’s product’. To test this claim a researcher asks 80 people about their liking for Acme. Assuming that the manufacturer is correct, find the probability that fewer than 53 prefer Acme. If 1000 researchers each questioned 80 people, how many of these researchers would be expected to record ‘fewer than 53 prefer Acme’ results?

10. Videos are packed in a box which contains 20 videos. 5% of the videos are faulty. The boxes are packed in crates which contain 50 boxes. Find the probabilities of the following events, clearly stating which distribution you are using and why.
   (a) A box contains 2 faulty videos.
   (b) A box contains at least 1 faulty video.
   (c) A crate contains between 35 and 39 (inclusive) boxes with at least 1 faulty video.
2. The Normal Distribution

**Miscellaneous exercise 2**

1. Given that $X \sim N(10, 2.25)$, find $P(X > 12)$. (OCR)

2. The random variable $X$ has the distribution $X \sim N(10, 8)$. Find $P(X > 6)$. (OCR)

3. $W$ is a normally distributed random variable with mean 0.58 and standard deviation 0.12. Find $P(W < 0.79)$. (OCR)

4. $X$ is a random variable with the distribution $X \sim N(140, 56.25)$. Find the probability that $X$ is greater than 128.75. (OCR)

5. The manufacturers of a new model of car state that, when travelling at 56 miles per hour, the petrol consumption has a mean value of 32.4 miles per gallon with standard deviation 1.4 miles per gallon. Assuming a normal distribution, calculate the probability that a randomly chosen car of that model will have a petrol consumption greater than 30 miles per gallon when travelling at 56 miles per hour. (OCR)

6. A normally distributed random variable, $X$, has mean 20.0 and variance 4.15. Find the probability that $18.0 < X < 21.0$. (OCR)

7. The lifetime of a Fotobrite light bulb is normally distributed with mean 1020 hours and standard deviation 85 hours. Find the probability that a Fotobrite bulb chosen at random has a lifetime between 1003 and 1088 hours. (OCR)

8. The area that can be painted using one litre of Luxibrite paint is normally distributed with mean 13.2 m$^2$ and standard deviation 0.197 m$^2$. The corresponding figures for one litre of Maxigloss paint are 13.4 m$^2$ and 0.343 m$^2$. It is required to paint an area of 12.9 m$^2$. Find which paint gives the greater probability that one litre will be sufficient, and obtain this probability. (OCR)

9. The time required to complete a certain car journey has been found from experience to have mean 2 hours 20 minutes and standard deviation 15 minutes.

   (a) Use a normal model to calculate the probability that, on one day chosen at random, the journey requires between 1 hour 30 minutes and 2 hours 40 minutes.

   (b) It is known that delays occur rarely on this journey, but that when they do occur they are lengthy. Give a reason why this information suggests that a normal distribution might not be a good model. (OCR)

10. The weights of eggs, measured in grams, can be modelled by a $X \sim N(85.0, 36)$ distribution. Eggs are classified as large, medium or small, where a large egg weighs 90.0 grams or more, and 25% of eggs are classified as small. Calculate

   (a) the percentage of eggs which are classified as large,

   (b) the maximum weight of a small egg. (OCR)

11. The random variable $X$ is normally distributed with mean and standard deviation both equal to $a$.

   Given that $P(X < 3) = 0.2$, find the value of $a$. (OCR)

12. The random variable $X$ is normally distributed with standard deviation 3.2. The probability that $X$ is less than 74 is 0.8944.

   (a) Find the mean of $X$.

   (b) Fifty independent observations of $X$ are made. Find the expected number of observations that are less than 74. (OCR)
13 A random variable $X$ has a $N(m,4)$ distribution. Its associated normal curve is shown in the diagram. Find the value of $m$ such that the shaded area is 0.800, giving your answer correct to 3 significant figures.

14 A machine cuts a very long plastic tube into short tubes. The length of the short tubes is modelled by a normal distribution with mean $m$ cm and standard deviation 0.25 cm. The value of $m$ can be set by adjusting the machine. Find the value of $m$ for which the probability is 0.1 that the length of a short tube, picked at random, is less than 6.50 cm. The machine is adjusted so that $m = 6.40$, the standard deviation remaining unchanged. Find the probability that a tube picked at random is between 6.30 and 6.60 cm long.

15 A university classifies its degrees as Class 1, Class 2.1, Class 2.2, Class 3, Pass and Fail. Degrees are awarded on the basis of marks which may be taken as continuous and modelled by a normal distribution with mean 57.0 and standard deviation 10.0. In a particular year, the lowest mark for a Class 1 degree was 70.0, the lowest mark for a Class 2.1 degree was 60.0, and 4.5% of students failed. Calculate
(a) the percentage of students who obtained a Class 1 degree,
(b) the percentage of students who obtained a Class 2.1 degree,
(c) the lowest possible mark for a student who obtained a Pass degree.

16 The number of hours of sunshine at a resort has been recorded for each month for many years. One year is selected at random and $H$ is the number of hours of sunshine in August of that year. $H$ can be modelled by a normal variable with mean 130.
(a) Given that $P(H < 179) = 0.975$, calculate the standard deviation of $H$.
(b) Calculate $P(100 < H < 150)$.

17 The mass of grapes sold per day in a supermarket can be modelled by a normal distribution. It is found that, over a long period, the mean mass sold per day is 35.0 kg, and that, on average, less than 15.0 kg are sold on one day in twenty.
(a) Show that the standard deviation of the mass of grapes sold per day is 12.2 kg, correct to 3 significant figures.
(b) Calculate the probability that, on a day chosen at random, more than 53.0 kg are sold.

18 An ordinary unbiased dice is thrown 900 times. Using a suitable approximation, find the probability of obtaining at least 160 sixes.

19 The random variable $X$ is normally distributed with mean $\mu$ and variance $\sigma^2$. It is given that $P(X > 81.89) = 0.010$ and $P(X < 27.77) = 0.100$. Calculate the values of $\mu$ and $\sigma$.

20 It is given that 40% of the population support the Gamboge Party. One hundred and fifty members of the population are selected at random. Use a suitable approximation to find the probability that more than 55 out of the 150 support the Gamboge Party.
21 Two firms, Goodline and Megadelay, produce delay lines for use in communications. The delay time for a delay line is measured in nanoseconds (ns).
   (a) The delay times for the output of Goodline may be modelled by a normal distribution with mean 283 ns and standard deviation 8 ns. What is the probability that the delay time of one line selected at random from Goodline’s output is between 275 and 286 ns?
   (b) It is found that, in the output of Megadelay, 10% of the delay times are less than 274.6 ns and 7.5% are more than 288.2 ns. Again assuming a normal distribution, calculate the mean and standard deviation of the delay times for Megadelay. Give your answers correct to 3 significant figures.

22 State conditions under which a binomial probability model can be well approximated by a normal model.

     \[ X \text{ is a random variable with the distribution } X \sim B(12,0.42). \]
   (a) Anne uses the binomial distribution to calculate the probability that \( X < 4 \) and gives 4 significant figures in her answer. What answer should she get?
   (b) Ben uses a normal distribution to calculate an approximation for the probability that \( X < 4 \) and gives 4 significant figures in his answer. What answer should he get?
   (c) Given that Ben’s working is correct, calculate the percentage error in his answer.

23 The diameters of corks for wine bottles of a particular kind are normally distributed with mean 1.90 cm. In order to be acceptable, the diameter must be between 1.80 cm and 2.00 cm. The proportion of corks that are acceptable is 90%. Calculate the standard deviation of the diameters.

A sample of \( n \) corks is selected. Give a condition under which the number of acceptable corks in the sample can be modelled by a binomial distribution.

Assuming this condition holds,
   (a) for the case \( n = 6 \), calculate the probability of obtaining exactly 5 acceptable corks,
   (b) for the case \( n = 80 \), use a suitable approximation to estimate the probability that 70 or fewer corks will be acceptable.

24 A large box contains many plastic syringes, but previous experience indicates that 10% of the syringes in the box are defective. 5 syringes are taken at random from the box. Use a binomial model to calculate, giving your answers correct to three decimal places, the probability that
   (a) none of the 5 syringes is defective,
   (b) at least 2 syringes out of the 5 are defective.

Discuss the validity of the binomial model in this context.

Instead of removing 5 syringes, 100 syringes are picked at random and removed. A normal distribution may be used to estimate the probability that at least 15 out of the 100 syringes are defective. Give a reason why it may be convenient to use a normal distribution to do this, and calculate the required estimate.
On average my train is late on 45 journeys out of 100. Next week I shall be making 5 train journeys. Let \( X \) denote the number of times my train will be late.

(a) State one assumption which must be made for \( X \) to be modelled by a binomial distribution.

(b) Find the probability that my train will be late on all of the 5 journeys.

(c) Find the probability that my train will be late on 2 or more out of the 5 journeys.

Approximate your binomial model by a suitable normal model to estimate the probability that my train is late on 20 or more out of 30 journeys.

The random variable \( Y \) has the distribution \( N(\mu, 16) \). Given that \( P(Y > 57.50) = 0.1401 \), find the value of \( \mu \) giving your answer correct to 2 decimal places.

The playing time, \( T \) minutes, of classical compact discs is modelled by a normal variable with mean 61.3 minutes. Calculate the standard deviation of \( T \) if 5% of discs have playing times greater than 78 minutes.

The random variable \( Y \) is such that \( Y \sim N(8, 25) \). Show that, correct to 3 decimal places, \( P(8 - 6.2 < Y < 8) = 0.785 \).

Three random observations of \( Y \) are made. Find the probability that exactly two observations will lie in the interval defined by \( |Y - 8| < 6.2 \).

It is estimated that, on average, one match in five in the Football League is drawn, and that one match in two is a home win.

(a) Twelve matches are selected at random. Calculate the probability that the number of drawn matches is

(i) exactly three,

(ii) at least four.

(b) Ninety matches are selected at random. Use a suitable approximation to calculate the probability that between 13 and 20 (inclusive) of the matches are drawn.

(c) Twenty matches are selected at random. The random variables \( D \) and \( H \) are the numbers of drawn matches and home wins, respectively, in these matches. State, with a reason, which of \( D \) and \( H \) can be better approximated by a normal variable.

Squash balls, dropped onto a concrete floor from a given point, rebound to heights which can be modelled by a normal distribution with mean 0.8 m and standard deviation 0.2 m. The balls are classified by height of rebound, in order of decreasing height, into these categories: Fast, Medium, Slow, Super-Slow and Rejected.

(a) Balls which rebound to heights between 0.65 m and 0.9 m are classified as Slow. Calculate the percentage of balls classified as Slow.

(b) Given that 9% of balls are classified as Rejected, calculate the maximum height of rebound of these balls.

(c) The percentages of balls classified as Fast and as Medium are equal. Calculate the minimum height of rebound of a ball classified as Fast, giving your answer correct to 2 decimal places.